A LINEAR STUDY OF HIGH-DRAG STATES AND FLOW STAGNATION PRODUCED BY MOUNTAIN WAVES

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Abstract: A linear model of gravity waves generated by stratified airflow over mountains is developed. The model provides simple, closed-form formulas for the surface drag in a situation where conditions for wave resonance exist. The wind is constant near the surface and decreases linearly above. The drag normalized by its value in the absence of shear is found to depend on two parameters: the height of the interface where the shear is discontinuous and the Richardson number, *Ri*, in the region above. This drag attains maxima when the height of the interface induces constructive interference between the upward and downward propagating reflected waves, and minima when there is destructive interference. The amplitude of the drag modulation becomes larger for lower *Ri*. It is also shown that, for *Ri*<2.25, the locations where wave breaking is first predicted to occur in flow over a 2D ridge become displaced horizontally and vertically by an amount depending on *Ri*.

Keywords *– Mountain waves, High-drag states, Resonance*

1. INTRODUCTION

Among the phenomena occurring in orographically generated gravity waves in the atmosphere, the problem of high-drag states is one of the most widely studied. This is presumably because the globally integrated pressure drag exerted by mountains on the atmosphere receives a large contribution from these situations. Since the drag must be parameterized in large-scale numerical models, studying the mechanisms behind this phenomenon is a problem of obvious practical, as well as fundamental, importance.

The most influential theories that attempt to explain high-drag states are those of Clark & Peltier (1984) and of Smith (1985). Clark & Peltier propose that the drag enhancement results from the reflection of gravity waves at environmental or self-generated critical levels. This reflection itself may be understood in the framework of linear resonance, leading to a spacing between consecutive high-drag states, in terms of the critical level height, of half the vertical wavelength of the gravity waves. Smith, on the other hand, proposes a totally nonlinear explanation, where the drag enhancement results from hydraulic behaviour of the flow, leading to a spacing of one vertical wavelength between consecutive critical level heights producing high-drag states.

Numerical simulations confirm that high-drag states are indeed strongly nonlinear, and that the best predictions result from the theory of Smith (1985). However, the recent study of Wang and Lin (1999) has suggested that some insights into this problem may be obtained using linear theory. They found that, in the linear regime, the key height determining the maximum surface response of the flow is, not the critical level, but the height where the shear has a discontinuity, in their idealized wind profile. However, Wang and Lin focused only on the velocity perturbations, and did not calculate the drag. This study (see also Teixeira et al. 2005) aims to calculate the drag in resonant situations using linear theory, in order to understand how linear processes may initiate high-drag states.

Linear theory is also used to show that, in the same situations, the critical conditions for wave breaking in flow over a 2D ridge may be different from those widely accepted (see also Teixeira and Miranda 2005).

2. THEORETICAL MODEL

The atmosphere is assumed to be stably stratified, with Brunt-Väisälä frequency N, in stationary motion and in hydrostatic equilibrium. If the equations of fluid mechanics are linearized with respect to the perturbations induced by flow over an isolated mountain, and combined, the hydrostatic Taylor-Goldstein equation results:

$$\hat{w}+\left(\frac{N^2(k_1^2+k_2^2)}{\left(Uk_1+Vk_2\right)^2}-\frac{U''k_1+V''k_2}{Uk_1+Vk_2}\right)\hat{w}=0\,, \tag{1}$$

where $\hat{w}$ is the Fourier transform of the vertical velocity perturbation, (U,V) is the unperturbed wind velocity, (k_1,k_2) is the horizontal wavenumber vector of the gravity waves and the primes denote differentiation with respect to z.

The simplest wind profile capable of generating gravity wave resonance is considered:

$$U=\begin{cases}U_0 & if\ z\leq z_1\\ U_0(z_c-z)/(z_c-z_1) & if\ z>z_1\end{cases}. \tag{2}$$

This profile, which near the surface is constant, with windspeed U_0, and above $z=z_1$ decreases linearly, reaching a critical level at $z=z_c$, induces wave reflection at $z=z_1$ and wave absorption at $z=z_c$, making the divergence of the windspeed towards infinity above $z=z_c$ essentially irrelevant for the behaviour of the gravity waves and of the drag near the surface.

Equation (1) must be solved subject to two boundary conditions: that the flow follows the topography at the surface,

$$\hat{w}(z=0)=iU_0k_1\hat{\eta}\,, \tag{3}$$

where $\hat{\eta}$ is the Fourier transform of the surface elevation, and that the wave energy radiates upward in the region
$z>z_1$ (this is known as the radiation boundary condition). Once $\hat{w}$ is determined in the whole domain following this procedure, the Fourier transform of the pressure perturbation may be calculated and, given the form the terrain elevation, the pressure drag on the mountain may be calculated.

When the drag is normalized by its value in the absence of shear, D_0, the final result in the case of flow over an axisymmetric mountain is

$$\frac{D}{D_0}=\frac{1}{\pi}\int_0^{2\pi}\frac{\cos^2\theta\left[1-(1/4)Ri^{-1}\cos^2\theta\right]^{1/2}}{1-(1/2)Ri^{-1/2}\cos\theta\sin\left[2Nz_1/(U_0\cos\theta)\right]}d\theta\,, \tag{4}$$

while in the case of flow over a 2D ridge, the normalized drag is given by

$$\frac{D}{D_0}=\frac{\left[1-(1/4)Ri^{-1}\right]^{1/2}}{1-(1/2)Ri^{-1/2}\sin\left(2Nz_1/U_0\right)}\,, \tag{5}$$

where $Ri=N^2(z_c-z_1)^2/U_0^2$ is the Richardson number in the shear layer ($z>z_1$).

These expressions show that the normalized drag only depends on two parameters: a dimensionless height formed with z_1, namely $Nz_1/\pi U_0$, and Ri. They will be compared with numerical simulations next.

3. RESULTS

3.1. Gravity wave drag

Figure 1 shows the variation of the normalized drag with the dimensionless height of the shear layer from (4) and (5) and from simulations of non-hydrostatic, nonlinear, mesoscale numerical models, albeit run for approximately linear and hydrostatic conditions. The agreement is quite good. For flow over a 2D ridge, the drag oscillates periodically, with a dimensionless period of 1, which when expressed in terms of z_1 corresponds to $\pi U_0/N$, or half the vertical wavelength of the gravity waves. Drag maxima occur at $Nz_1/\pi U_0$=0.25+n, where n is an integer, while drag minima occur at $Nz_1/\pi U_0$=0.75+n. In flow over an axisymmetric mountain, the drag also oscillates with $Nz_1/\pi U_0$, but this oscillation is only approximately periodic. Its amplitude decreases as $Nz_1/\pi U_0$ increases, due to wave dispersion.

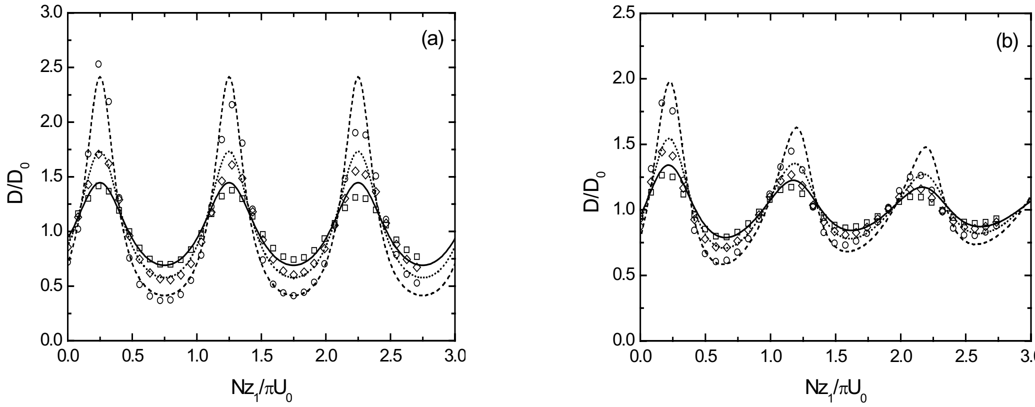

Figure 1. Normalized drag as a function of $Nz_1/\pi U_0$ from (4) and (5) (lines) and from numerical simulations (symbols). Solid line and squares: Ri=2, dotted line and diamonds: Ri=1, dashed line and circles: Ri=0.5. (a) 2D mountain ridge, (b) axisymmetric mountain

For both types of mountain, the amplitude of the drag modulation increases as Ri decreases. The observed behaviour is, in certain aspects, reminiscent of high-drag states, as observed in numerical simulations (e.g. Bacmeister and Pierrehumbert 1988, Miranda and Valente 1997), with the difference that the key height is z_1 instead of that generally assumed: z_c. The spacing of the drag maxima is also half of that observed in nonlinear numerical simulations of flow over a 2D ridge.

3.2. Flow structure

The behaviour of the drag is better understood when the flow structure is analysed. Figure 2 shows the streamwise velocity perturbation (shaded contours) and the isentropes (lines of constant total potential temperature, which may be regarded as streamlines since the flow is adiabatic) for flow over a 2D ridge. A high-drag state, corresponding to the third maximum of Fig. 1(a) and a low-drag state, corresponding to the third minimum, are illustrated.

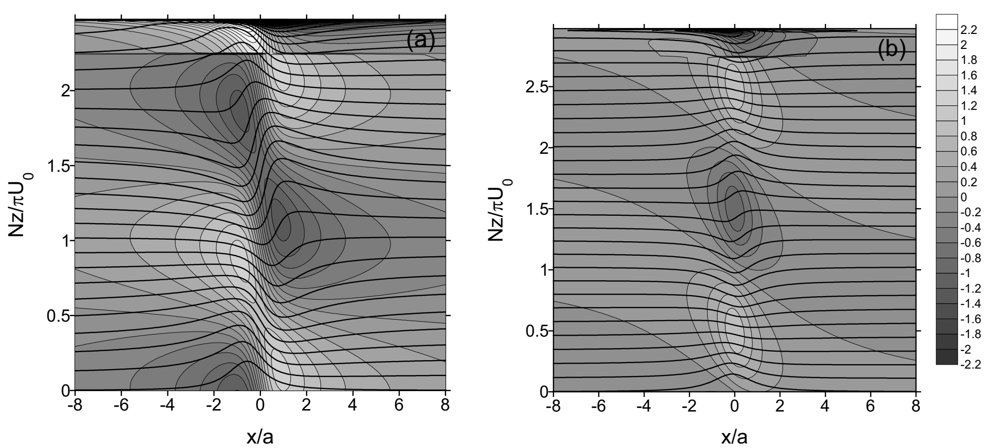

Figure 2. Normalized streamwise velocity perturbation (shaded contours) and isentropes (thick lines, with a spacing of 1K), from linear theory. (a) $Nz_1/\pi U_0$=2.25 and Ri=0.5, (b) $Nz_1/\pi U_0$=2.75 and Ri=0.5

It may be seen that the extrema of the streamwise velocity perturbation are much more intense in a high-drag situation than in a low-drag situation. These extrema also possess two-lobes, one upstream and another downstream of the mountain. This reinforces the asymmetry of the flow relative to the mountain, and is associated with an enhancement of the drag (in fact, in the constant-wind layer, the normalized streamwise velocity perturbation and the normalized pressure perturbation have the same value, but opposite signs). In Fig. 2(b), on the other hand, the flow perturbations are less intense and less asymmetric, which corresponds to lower drag. These differences are also reflected in the slope of the streamlines above the mountain, which is much gentler in Fig. 2(b) than in Fig. 2(a). The appearance of the flow field in both cases is caused by reflection of the gravity waves at $z=z_1$. Fig 2(a) corresponds to constructive interference between waves whose energy propagates upward and waves whose energy propagates downward. Fig 2(b) corresponds to destructive interference.

3.3. Wave breaking

Although the use of a linear model to predict wave breaking (or equivalently, flow stagnation) is questionable, because this is a highly nonlinear phenomenon by definition, the study of Smith (1989) has shown that this approach may have some qualitative value.

From the same linear framework as used previously, it is possible, for flow over a 2D ridge, to derive analytical expressions for the locations where flow stagnation first occurs. Figure 3 presents these results for the case of drag maxima (the situation most favourable for flow stagnation). For $Ri \geq 2.25$, linear theory predicts that flow stagnation occurs exactly above the ridge top and at a dimensionless height $Nz/\pi U_0=1.5$, a well-known result. However, as Ri decreases to 0.25, the location where flow stagnation first occurs moves downstream and downward towards $x/a=1$ and $Nz/\pi U_0=1$. This result is relevant for the problem of high-drag states, since flow stagnation generates critical levels, which appear to be important in nonlinear conditions.

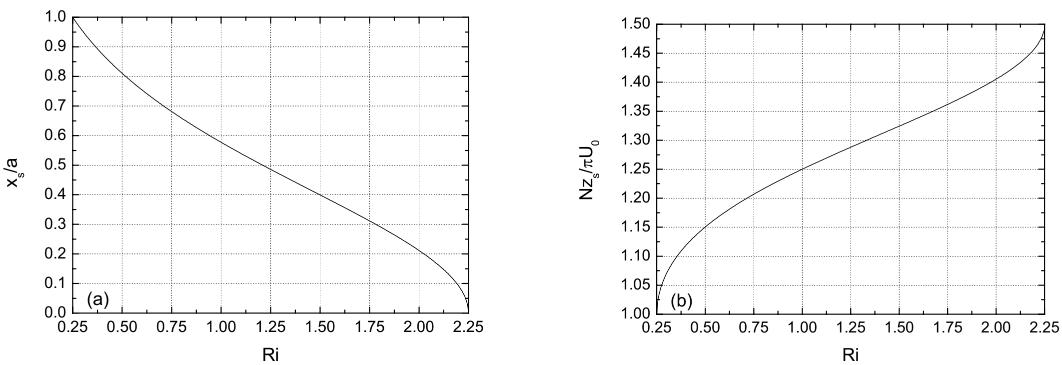

Figure 3. Normalized position of the lowest flow stagnation zone as a function of Ri. (a) horizontal, (b) vertical

4. CONCLUSION

The present study uses a linear model to address the problems of drag enhancement and flow stagnation in resonant gravity waves generated by stratified airflow over orography. The model suggests, in agreement with Wang and Lin (1999), that the key height determining the occurrence of high or low drag is that where the shear in the wind profile is discontinuous. It also suggests that the amplitude of the drag modulation increases as the Richardson number in the shear layer decreases. Preliminary nonlinear results (not shown) indicate that the dependence of the drag on Ri weakens considerably, and that the drag starts to behave quite differently in flow over a 2D ridge. While the linear predictions for the location of the drag maxima (see Fig. 2(b)) are still reasonably good in nonlinear flow over an axisymmetric mountain, in nonlinear flow over a 2D ridge these predictions (see Fig. 2(a)) are not adequate. The second drag maximum disappears and the remaining maxima are shifted towards higher $Nz_1/\pi U_0$ by a considerable amount. In that situation, only the theory of Smith (1985) provides adequate predictions. The reason why linear theory works better for axisymmetric (or generally 3D) mountains may be because wave dispersion enables the flow to remain closer to linear in this situation.

Since the theory of Smith has the critical level as a key height, its ability to predict nonlinear 2D flow suggests that, as the flow becomes more nonlinear, the key height governing the dynamics of high-drag states changes from z_1 to z_c. It would be interesting to understand how this transition occurs, but that is outside the scope of the present study.

For mountains of sufficiently high amplitude, wave breaking (or flow stagnation) happens, which leads to self-generated critical levels. The present study suggests that the locations where flow stagnation first occurs depend on *Ri*, which may have implications for the interpretation of high-drag states in highly nonlinear conditions.

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